EE 230
Lecture 15

Basic Applications of Operational Amplifiers

- Differential Amplifiers
- Impedance Converters

Nonideal Op Amp Characteristics (if time permits)

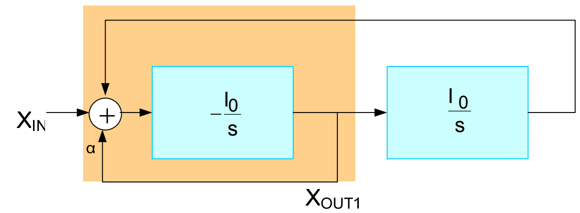
Applications of integrators to filter design

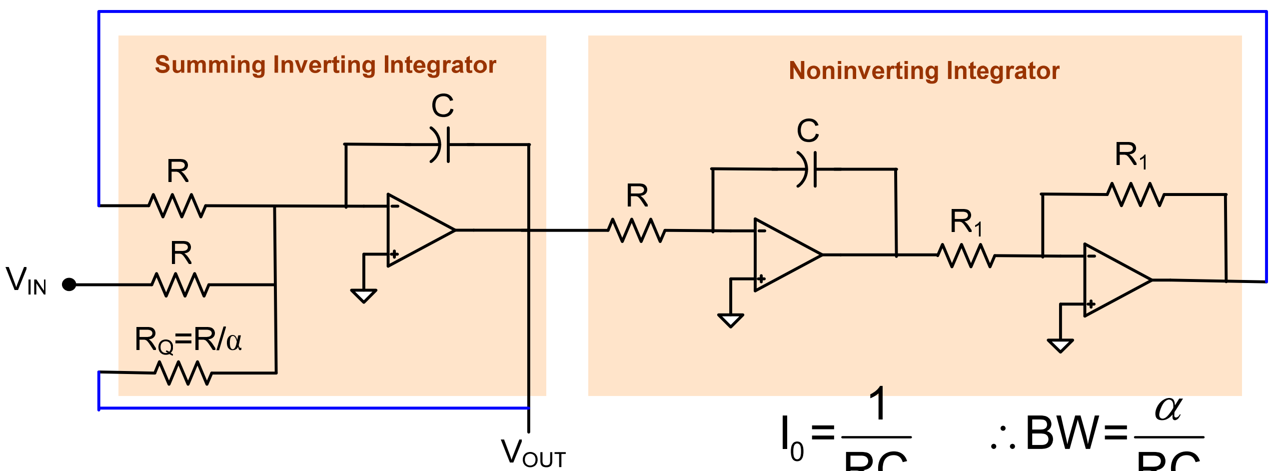
The 2nd order Bandpass Filter

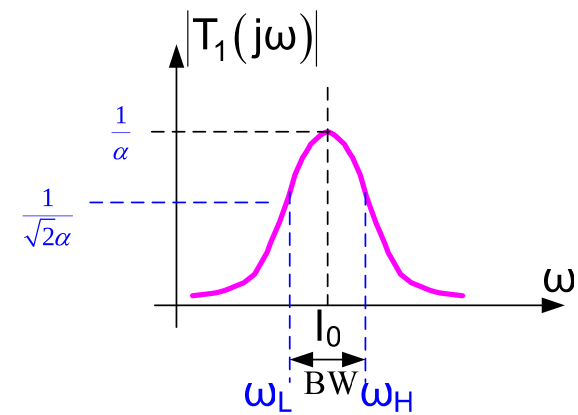
Design Strategy

Assume BW and ω_P are specified

$$T_{BP}(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2}$$







$$I_0=\frac{1}{RC} \qquad \therefore BW=\frac{\alpha}{RC}$$

$$\omega_P = I_0$$

1. Pick C (use some practical or convenient value)
2. Solve expression $\omega_P=\frac{1}{RC}$ to obtain R
3. Solve expression $BW=\frac{\alpha}{RC}$ to obtain α and thus R_Q

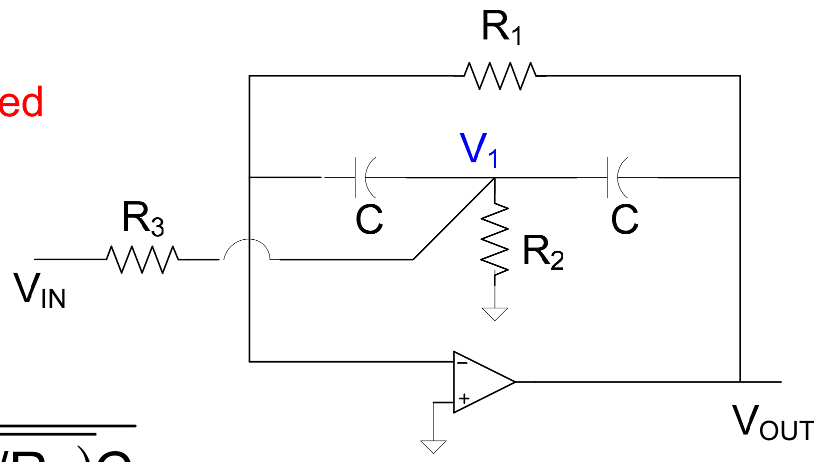
Review from Last Time

Another 2nd-order Bandpass Filter

Design Strategy

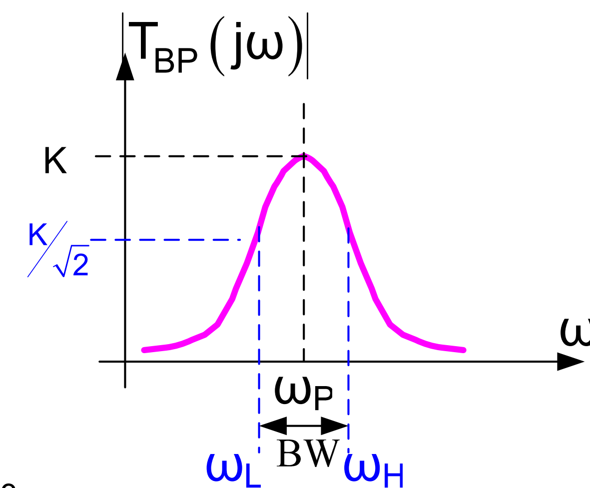
Assume BW, ω_P , and K are specified

$$T(s) = -\frac{\frac{s}{R_3C}}{s^2+s\left(\frac{2}{R_1C}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$



$$BW = \frac{2}{R_1C} \qquad \omega_P = \frac{1}{\sqrt{R_1(R_2//R_3)C}}$$

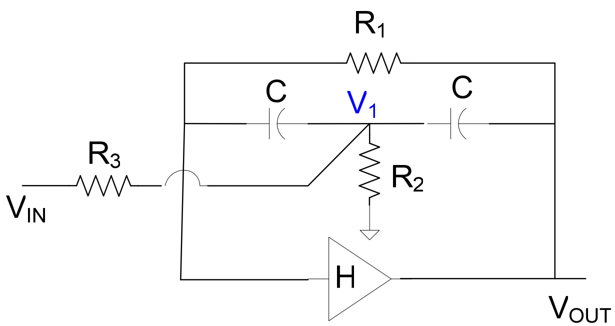
$$K = \frac{R_1}{2R_3}$$



1. Pick C to some practical or convenient value
2. Solve expression $BW = \frac{2}{R_1C}$ to obtain R_1
3. Solve expression $K = \frac{R_1}{2R_3}$ to obtain α and thus R_3
4. Solve expression $\omega_P = \frac{1}{\sqrt{R_1(R_2//R_3)C}}$ to obtain R_2

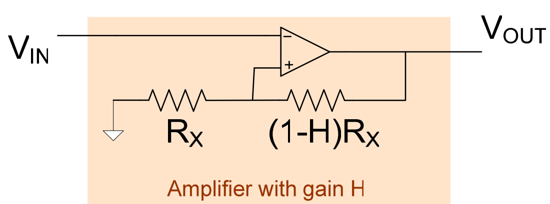
Review from Last Time

STAR 2nd-order Bandpass Filter

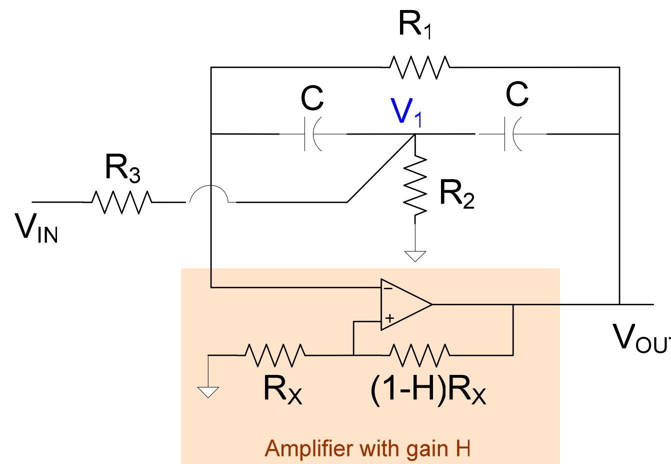


$$T(s) = -\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

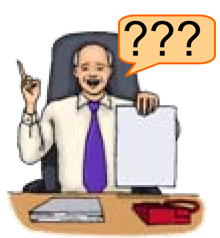
Implementation:



If op amp ideal, $\frac{V_{OUT}}{V_{IN}} = H$



Works fine !



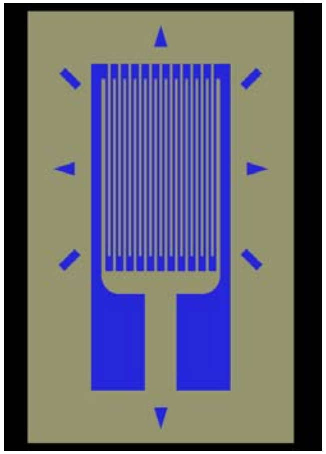
Will discuss why this happens later!

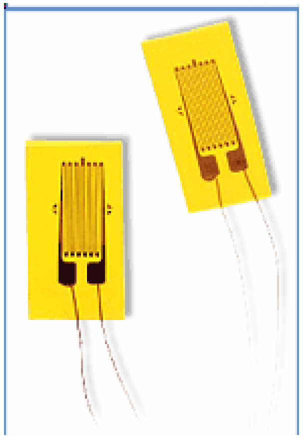
Reduces to previous bandpass filter at H gets large

Note that the “H” amplifier has feedback to positive terminal

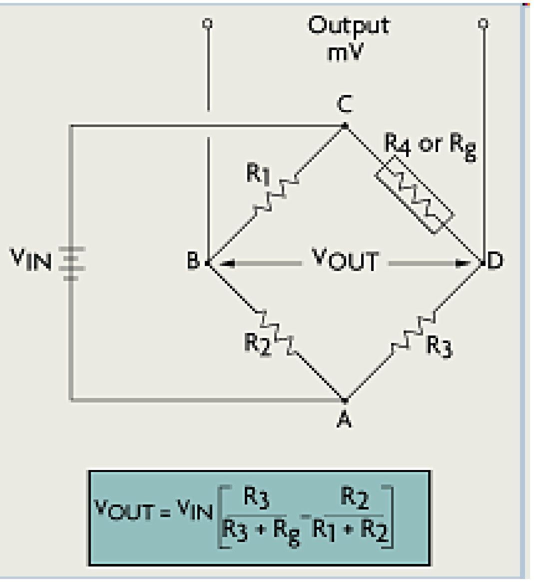
Differential Amplifiers

Many applications exist for difference (differential) amplifiers
Differential amplifiers are widely used
Strain gage is one application that demonstrates some challenges





Metal foil strain gages



Wheatstone Bridge
http://www.omega.com/prodinfo/straingages.html

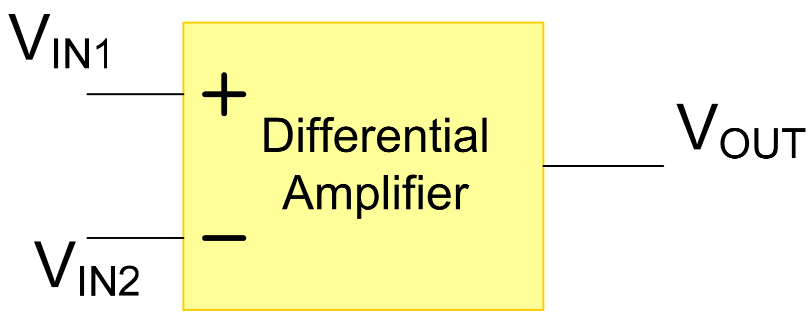
http://en.wikipedia.org/wiki/Strain_gauge

Assume V_A is ground

V_{OUT} is very small compared to V_B and V_D

Differential Amplifiers

Ideal differential amplifier



$$V_{OUT} = A\left(V_{IN1} - V_{IN2}\right)$$

$R_{OUT}=0$

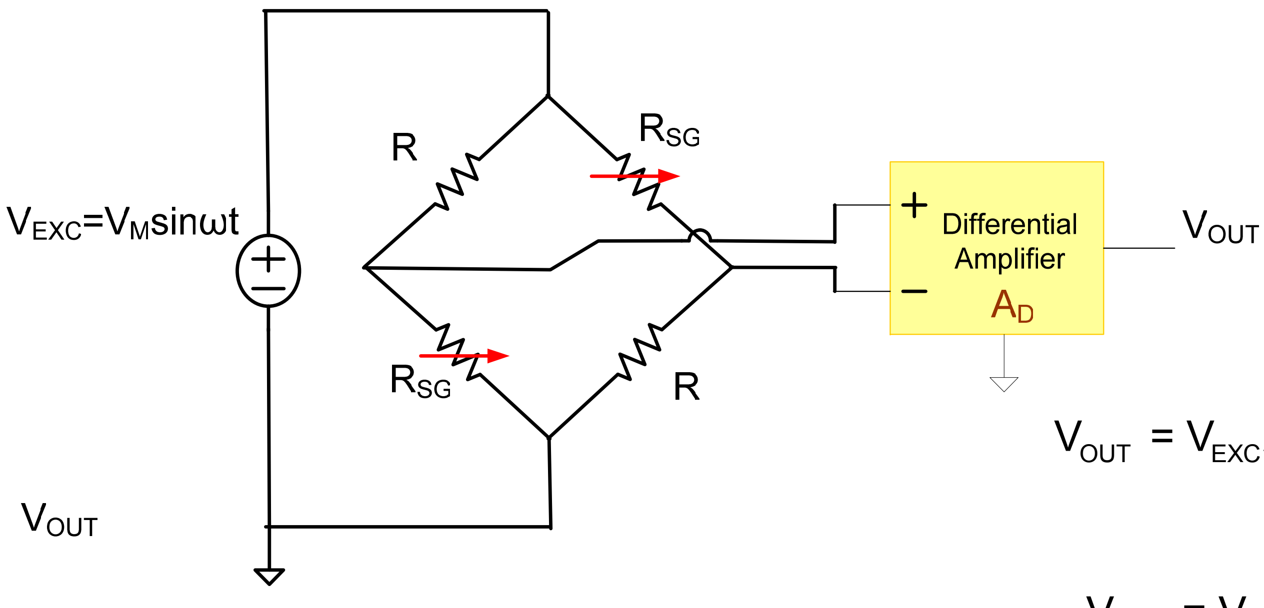
All $R_{IN}=\infty$

Ideally the output is not dependent upon the size of V_{IN1} or V_{IN2} but only upon their difference

This creates a challenge when designing differential amplifiers

Differential Amplifiers

Differential amplifier application



$$V_{OUT} = V_{EXC} A_D \left(\frac{R_{SG}}{R+R_{SG}} - \frac{R}{R+R_{SG}} \right)$$

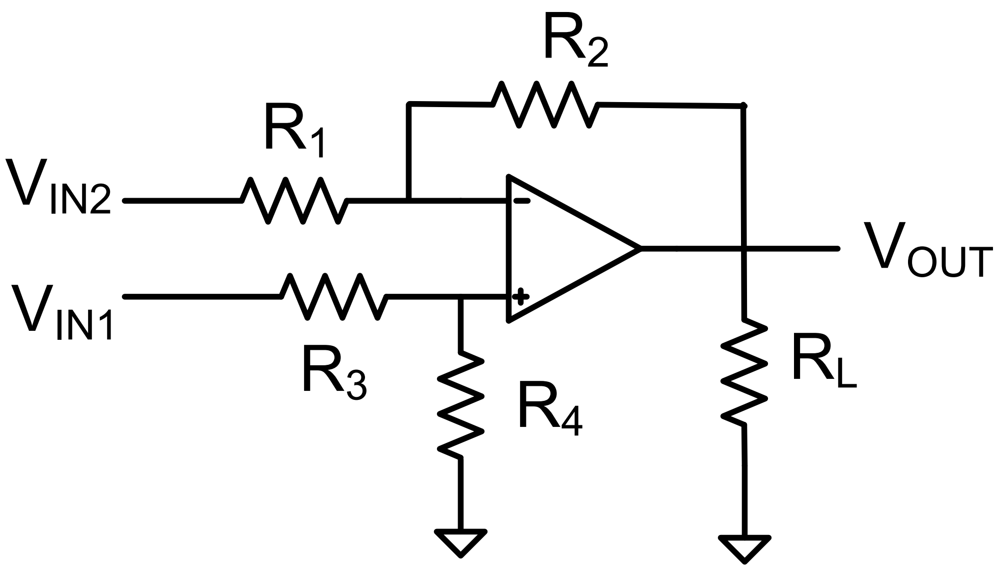
$$V_{OUT} = V_{EXC} A_D \left(\frac{R_{SG} - R}{R+R_{SG}} \right)$$

Let $R_{SG} = R + \Delta R$

$$V_{OUT} = V_{EXC} A_D \left(\frac{\Delta R}{2R+\Delta R} \right) \simeq V_{EXC} A_D \left(\frac{\Delta R}{2R} \right)$$

If ΔR is very small and varies linearly with strain, V_{OUT} varies linearly with strain

Differential Amplifiers



$$V_{OUT} = V_{IN1}\left(\frac{R_4}{R_3+R_4}\left[1+\frac{R_2}{R_1}\right]\right) - V_{IN2}\left(\frac{R_2}{R_1}\right)$$

$$V_{OUT} = V_{IN1}\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right) - V_{IN2}\left(\frac{R_2}{R_1}\right)$$

If $R_1=R_3$ and $R_2=R_4$, this becomes

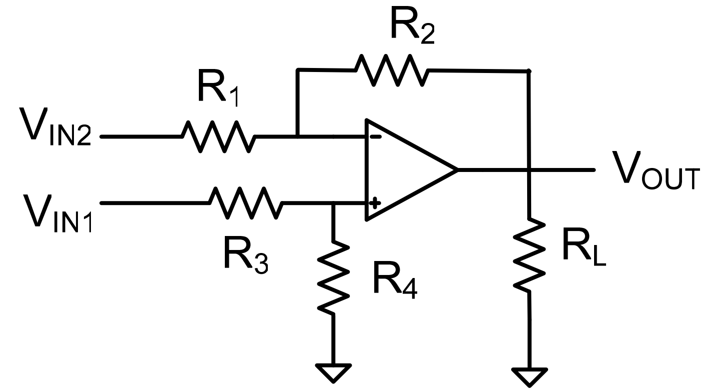
$$V_{OUT} = V_{IN1}\left(\frac{R_2}{R_1}\right) - V_{IN2}\left(\frac{R_2}{R_1}\right)$$

$$V_{OUT} = \frac{R_2}{R_1}\left(V_{IN1}-V_{IN2}\right)$$

Good matching is required to eliminate the dependence on V_{IN1} and V_{IN2}

Differential Amplifiers

Common Mode and Difference Mode Gains



$$V_{OUT}=V_{IN1}\left(\frac{R_4}{R_3+R_4}\left[1+\frac{R_2}{R_1}\right]\right)-V_{IN2}\left(\frac{R_2}{R_1}\right)$$

Define

$$V_{INC}=\frac{V_{IN1}+V_{IN2}}{2} \qquad V_{IN2}=V_{INC}-\frac{V_{IND}}{2}$$

These can be expressed as

$$V_{IND}=V_{IN1}-V_{IN2} \qquad V_{IN1}=V_{INC}+\frac{V_{IND}}{2}$$

$$V_{OUT}=\left(V_{INC}+\frac{V_{IND}}{2}\right)\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right)-\left(V_{INC}-\frac{V_{IND}}{2}\right)\left(\frac{R_2}{R_1}\right)$$

$$V_{OUT}=V_{INC}\left(\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right)-\left(\frac{R_2}{R_1}\right)\right)+V_{IND}\left(\frac{1}{2}\right)\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}+\frac{R_2}{R_1}\right)$$

Define

$$A_{COM}=\left(\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right)-\left(\frac{R_2}{R_1}\right)\right) \qquad A_{DIFF}=\frac{1}{2}\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}+\frac{R_2}{R_1}\right)$$

It follows that

$$V_{OUT}=V_{INC}A_{COM}+V_{IND}A_D$$

R_{IN} on each terminal is R_3+R_4

If $R_1=R_3$ and $R_2=R_4$

$$A_{COM}=0 \qquad A_{DIFF}=\frac{R_2}{R_1}$$

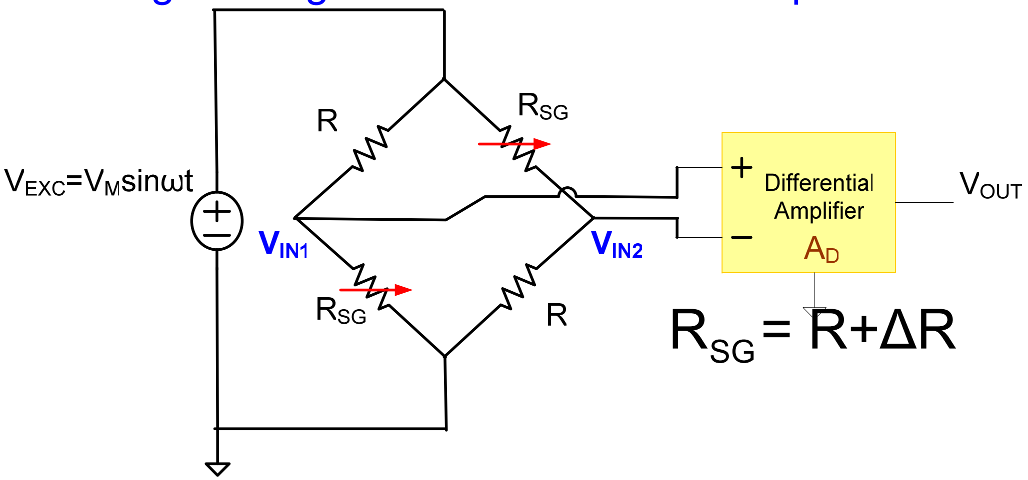
$$V_{OUT}=A_{DIFF}V_{IND}$$

Good matching is required to eliminate V_{INC}

Not easy to adjust or trim the gain

Differential Amplifiers

Example: Consider the performance of the bridge structure driving the differential amplifier where A_D is nominally 10,000. Neglect loading of bridge with the differential amplifier



$R_{SG} = R+\Delta R$

Assume $\Delta R=.00005R$

$V_{EXC}=4\sin\omega t$

$$V_{IN1} = V_{EXC}\left(\frac{R_{SG}}{R+R_{SG}}\right) \simeq \frac{V_{EXC}}{2}\left(1+\frac{\Delta R}{R}\right)$$

$$V_{IN2} = V_{EXC}\left(\frac{R}{R+R_{SG}}\right) \simeq \frac{V_{EXC}}{2}\left(1-\frac{\Delta R}{R}\right)$$

$V_{IN1}=2.0001\sin\omega t \qquad V_{IN2}=1.9999\sin\omega t$

$$V_{IND}=V_{IN1}-V_{IN2}=.0002\sin\omega t$$

$$V_{INC}=\frac{V_{IN1}+V_{IN2}}{2} \simeq 2\sin\omega t$$

All signal information is carried in the difference signal V_{IND}

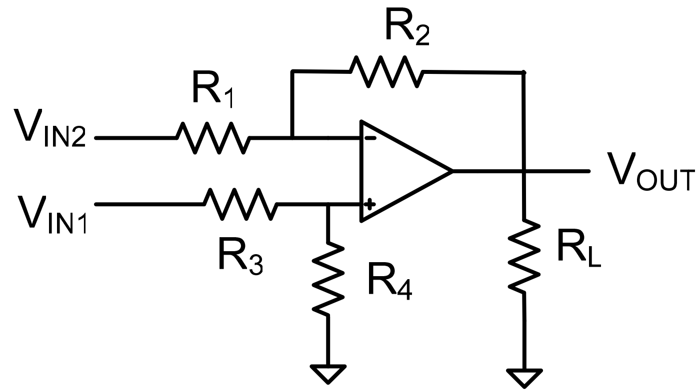
But observe $V_{IND} \ll V_{INC}$

Differential Amplifiers

Example (cont)

V_{IN1}=2.0001sinωt V_{IN2}=1.9999sinωt

V_{INC}=2 sinωt V_{IND}=.0002sinωt



Case 1 Matched resistors

If $R_1=R_3$ and $R_2=R_4$

$$V_{OUT} = A_{DIFF} V_{IND}$$

$$V_{OUT}=V_{INC}\left(\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right)-\left(\frac{R_2}{R_1}\right)\right)+V_{IND}\left(\frac{1}{2}\right)\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}+\frac{R_2}{R_1}\right)$$

$$A_{DIFF}=\frac{R_2}{R_1} \qquad A_{COM}=0 \qquad \frac{R_2}{R_1}=10000$$

Ideally $R_1=R_3$ and $R_2=R_4$

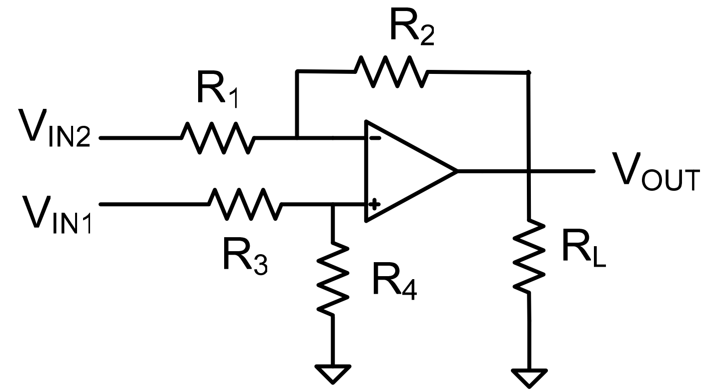
$$V_{OUT} = 10000 \bullet .0002\sin\omega t = 2\sin\omega t$$

Differential Amplifiers

Example (cont)

V_{IN1}=2.0001sinωt V_{IN2}=1.9999sinωt

V_{INC}=2 sinωt V_{IND}=.0002sinωt



Case 1 Matched resistors

$$V_{OUT} = 2\sin\omega t$$

$$V_{OUT}=V_{INC}\left(\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right)-\left(\frac{R_2}{R_1}\right)\right)+V_{IND}\left(\frac{1}{2}\right)\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}+\frac{R_2}{R_1}\right)$$

Ideally $R_1=R_3$ and $R_2=R_4$

Case 2 $R_1=R_3$, $R_4=1.01R_2$ $\frac{R_2}{R_1}=10000$

$$V_{OUT}=V_{INC}\left(\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}\right)-\left(\frac{R_2}{R_1}\right)\right)+V_{IND}\left(\frac{1}{2}\right)\left(\frac{R_1+R_2}{R_3+R_4}\bullet\frac{R_4}{R_1}+\frac{R_2}{R_1}\right)$$

$$V_{OUT}=V_{INC}\left(\left(\frac{10001R_1}{10101R_1}\bullet\frac{1.01\bullet 10000R_1}{R_1}\right)-(10000)\right)+V_{IND}\left(\frac{1}{2}\right)\left(\left(\frac{10001R_1}{10101R_1}\bullet\frac{1.01\bullet 10000R_1}{R_1}\right)+10000\right)$$

$$V_{OUT}=V_{INC}(.0099)+V_{IND}(10000.005)$$

$$V_{OUT}=2\sin\omega t(.0099)+(.0002\sin\omega t)(10000.005)$$

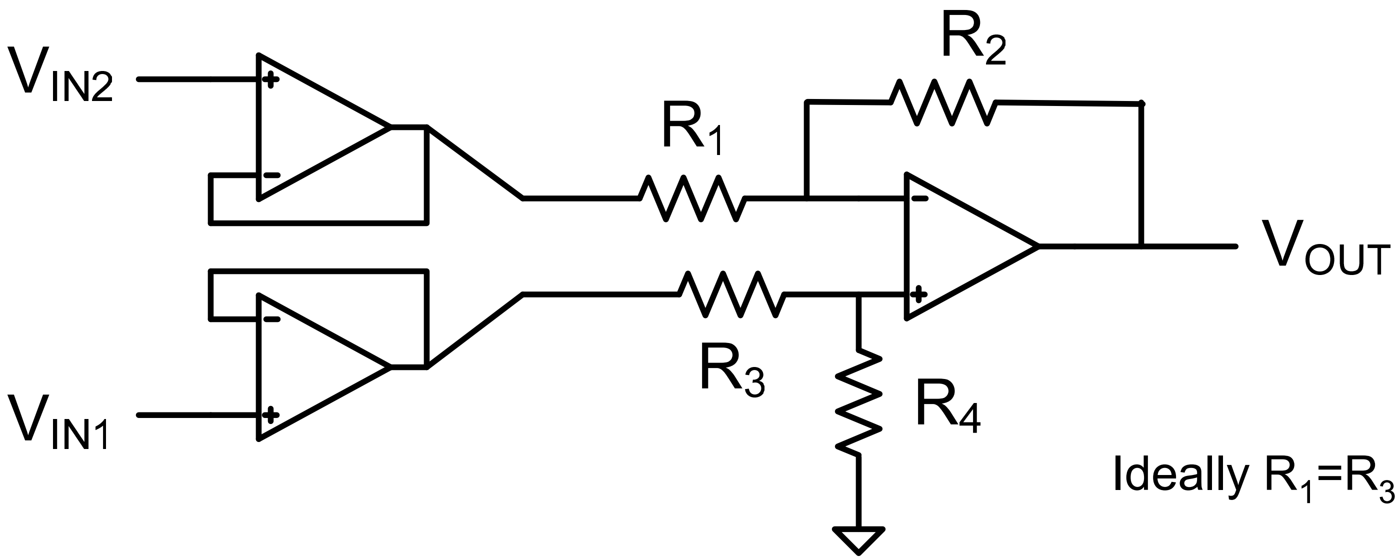
$$V_{OUT}=\sin\omega t(.0198)+(\sin\omega t)(2.000001)$$

$$V_{OUT}=(\sin\omega t)(2.019801)$$

Note a 1% error in a resistor causes a 2% error in output

Differential Amplifiers

Buffered Difference Amplifier



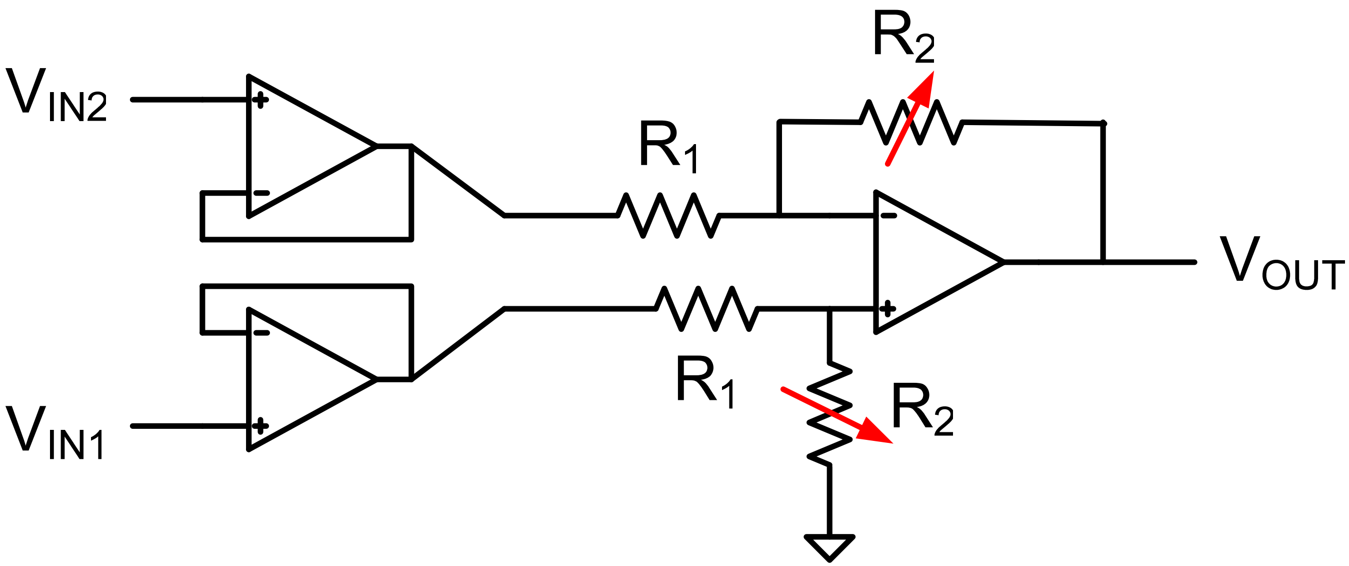
Ideally R_1=R_3 and R_2=R_4

Minimizes loading effect of difference amplifier on source

Not easy to adjust or trim the gain

Differential Amplifiers

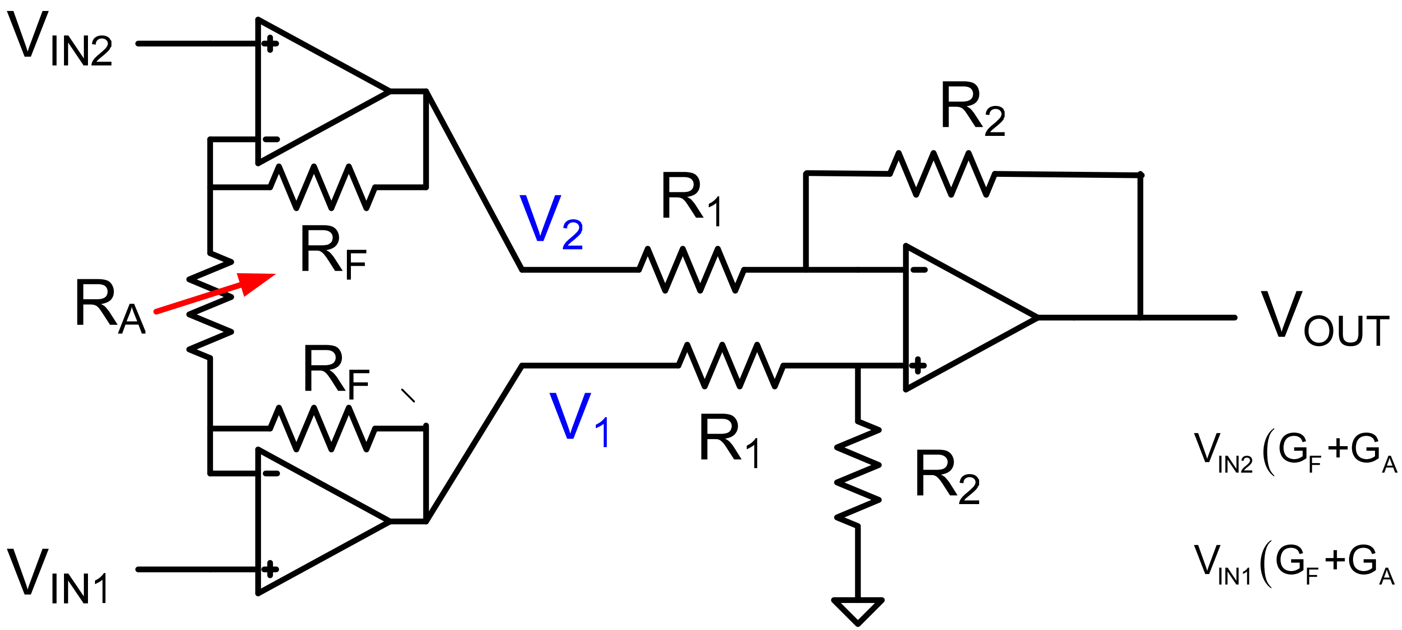
Buffered Difference Amplifier



Not easy to adjust or trim the gain

Differential Amplifiers

Instrumentation Amplifier



$$V_{IN2}\left(G_F + G_A\right) = G_F V_2 + G_A V_{IN1}$$

$$V_{IN1}\left(G_F + G_A\right) = G_F V_1 + G_A V_{IN2}$$

$$\left(V_{IN2} - V_{IN1}\right)\left(G_F + G_A\right) = G_F\left(V_2 - V_1\right) + G_A\left(V_{IN1} - V_{IN2}\right)$$

$$\left(V_{IN2} - V_{IN1}\right)\left(G_F + 2G_A\right) = G_F\left(V_2 - V_1\right)$$

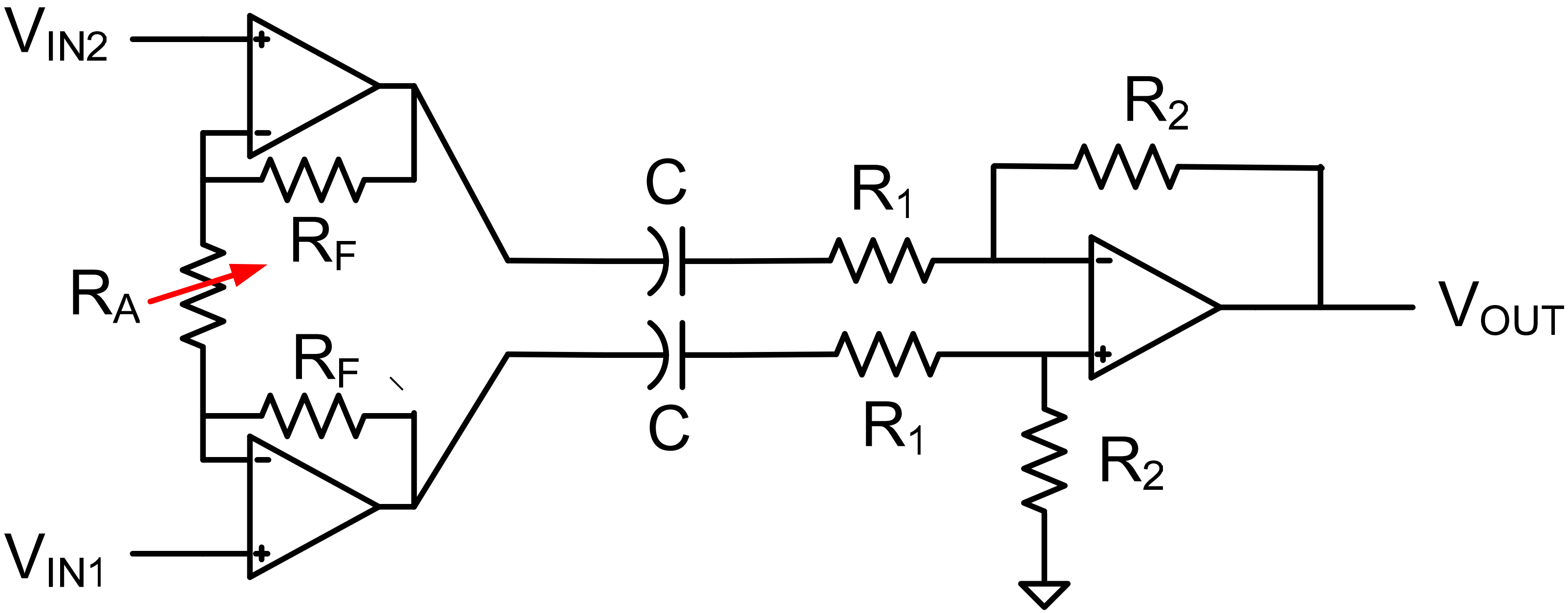
$$\left(V_{IN2} - V_{IN1}\right)\left(1 + 2\frac{R_F}{R_A}\right) = \left(V_2 - V_1\right)$$

$$V_{OUT} = \left(V_{IN2} - V_{IN1}\right)\left(1 + 2\frac{R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)$$

- Gain can be adjusted with the single resistor R_A
- Gain adjustment does not affect trimming of R_1 or R_2
- Input impedance is infinite
- Can provide higher gains

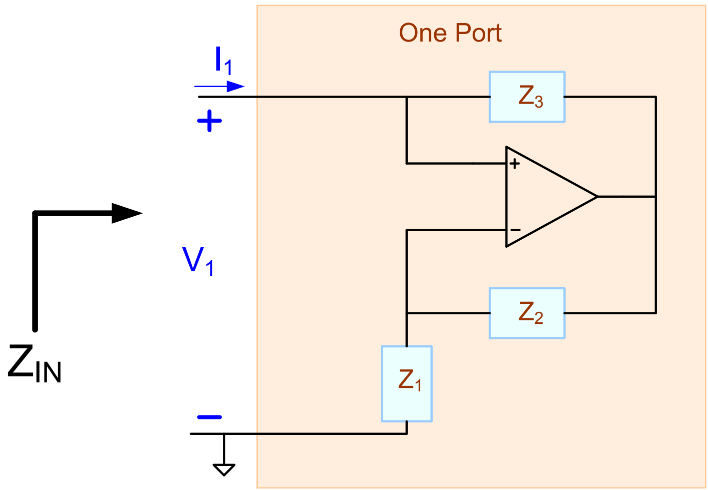
Differential Amplifiers

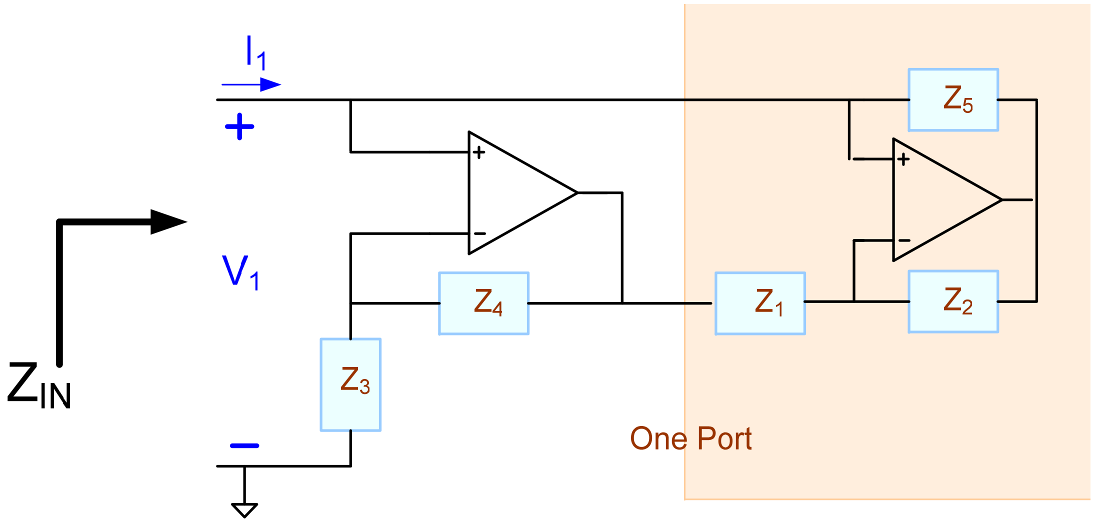
Instrumentation Amplifier



- Can reduce effects of dc offset if gain must be very large
- Must pick C to that frequencies of interest are in passband

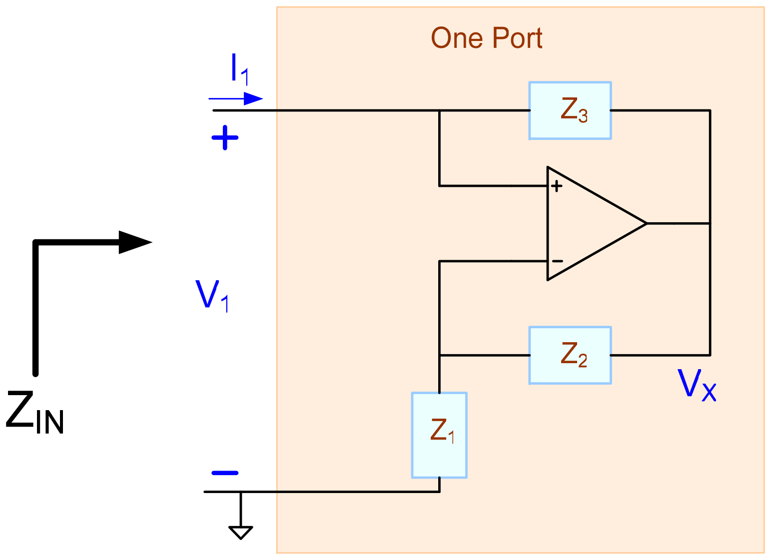
Impedance Converters





Note these circuits are strictly one-ports and have no output node

Impedance Converters

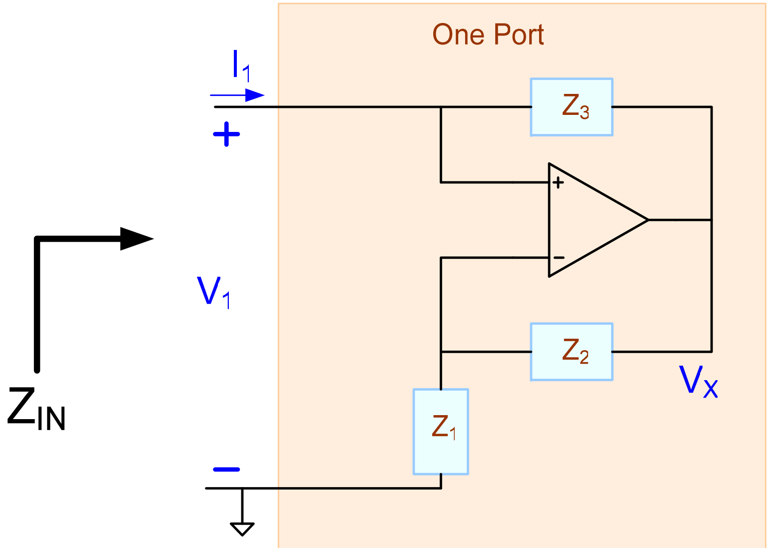


$$\left.\begin{aligned} V_1(G_1+G_2) &= V_X G_2 \\ I_1 &= (V_1-V_X)G_3 \end{aligned}\right\}$$

$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

Observe this input impedance is negative!

Impedance Converters



$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

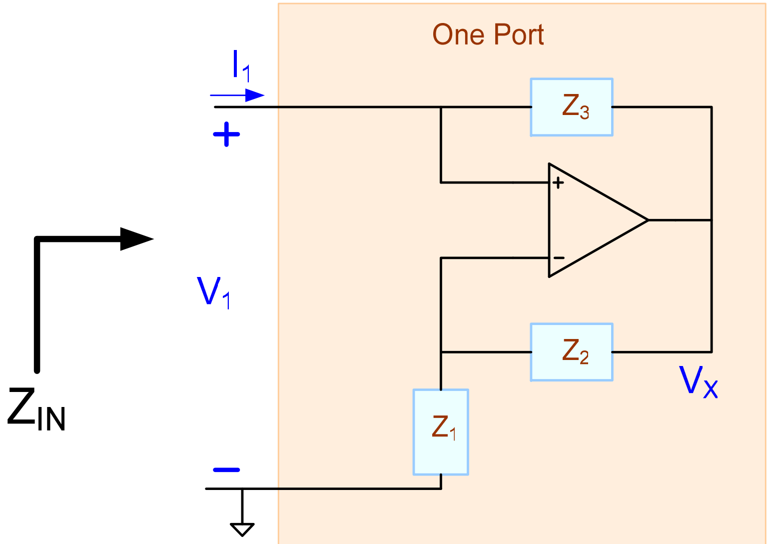
If $Z_1=R_1$, $Z_2=R_2$ and $Z_3=R_3$, $$Z_{IN} = -\frac{R_1 R_3}{R_2}$$ This is a negative resistor !

If $Z_2=1/sC$, $Z_1=R_1$ and $Z_3=R_3$, $$Z_{IN} = -sCR_1R_3$$ This is a negative inductor !

If $Z_2=R_2$, $Z_1=1/sC$ and $Z_3=R_3$, $$Z_{IN} = -\frac{R_3}{sCR_2}$$ This is a negative capacitor !

This is termed a Negative Impedance Converter

Impedance Converters



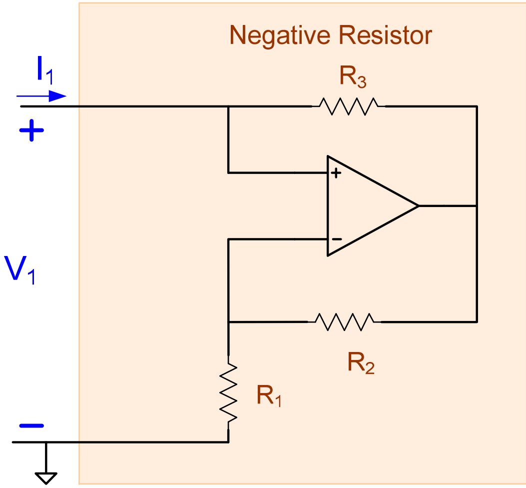
$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

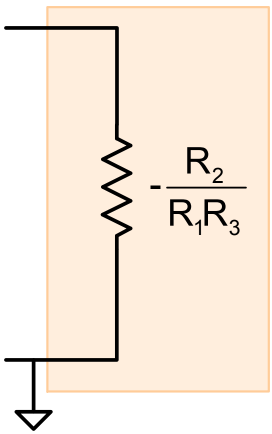
If $Z_2 = 1/sC$, $Z_1 = R_1$ and $Z_3 = R_3$, $\quad Z_{IN} = -sCR_1R_3$

Modification of NIC to provide a positive inductance:

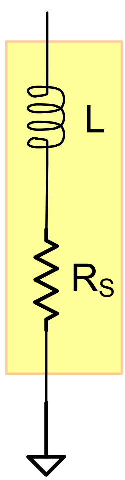
Replace Z_1 itself with a second NIC that has a negative input impedance

Negative Impedance Converter

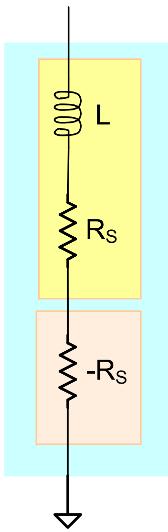




One application of NIC



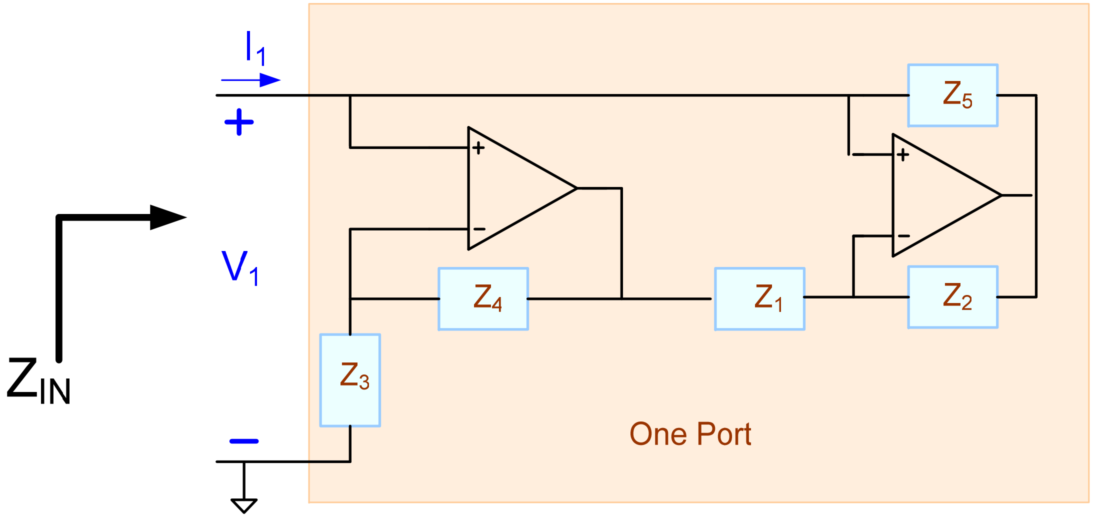
Lossy Inductor



If select components so that $R_S = \frac{R_2}{R_1 R_3}$

Lossless Inductor

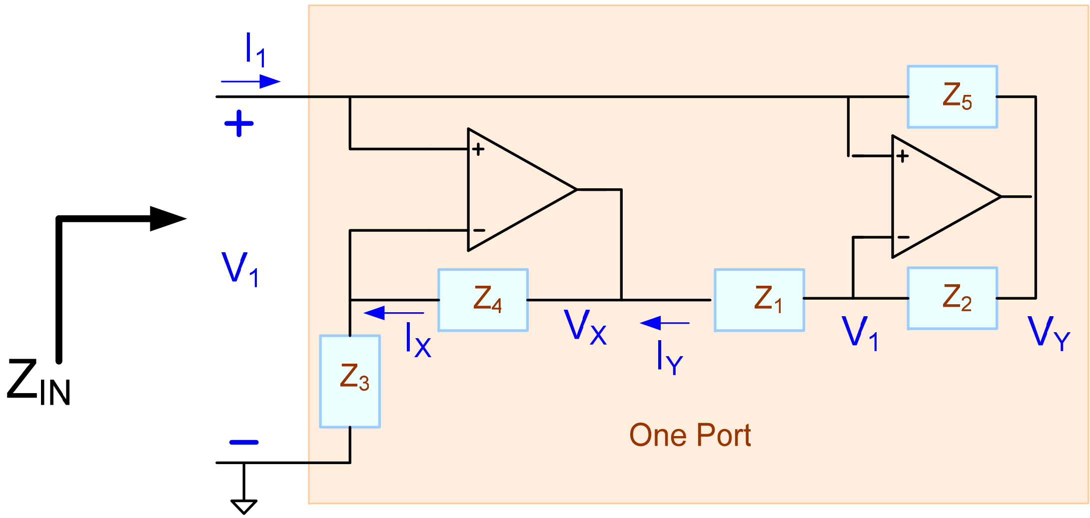
Impedance Converters



$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

This circuit is often called a Gyrator

Gyrator Analysis



$$I_X = V_1 G_3$$

$$V_X = V_1 + V_1 G_3 / G_4 \;= V_1\left(1 + \frac{G_3}{G_4}\right)$$

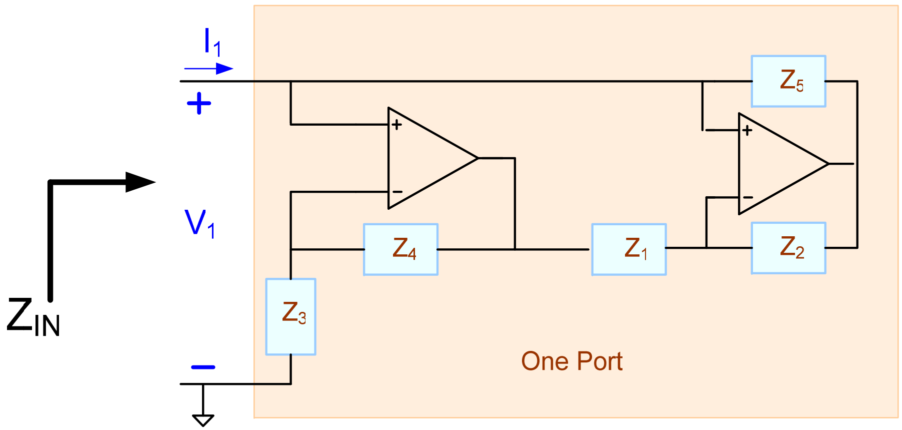
$$I_Y = (V_1 - V_X) G_1 \quad = V_1\left(-\frac{G_3}{G_4}\right) G_1$$

$$V_Y = V_1 + I_Y / G_2 \;= V_1\left(1 - \frac{G_3}{G_4}\left(\frac{G_1}{G_2}\right)\right)$$

$$I_1 = (V_1 - V_Y) G_5 = V_1\left(\frac{G_3}{G_4}\left(\frac{G_1}{G_2}\right)\right) G_5$$

$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

Gyrator Applications



$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

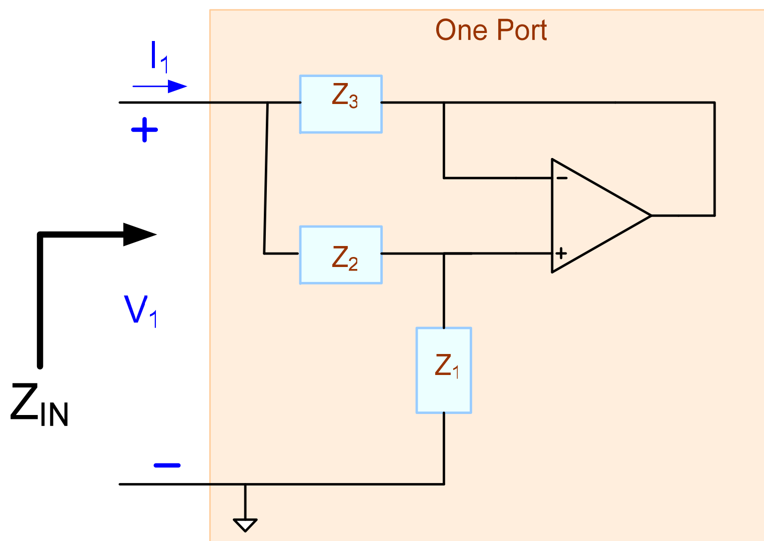
If $Z_1 = Z_3 = Z_4 = Z_5 = R$ and $Z_2 = 1/sC$ $\quad Z_{IN} = \left(R^2 C\right)s \quad$ This is an inductor of value $L = R^2 C$

If $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = R_5$ and $Z_1 = 1/sC$ $\quad Z_{IN} = \dfrac{R_3 R_5}{sCR_2 R_4}$

This is a capacitor of value $\quad C_{EQ} = C\dfrac{R_2 R_4}{R_3 R_5} \quad$ (can scale capacitance up or down)

If $Z_2 = Z_4 = Z_5 = R$ and $Z_1 = Z_3 = 1/sC$ $\quad Z_{IN} = \left(R^3 C^2\right)s^2 \quad$ This is a “super” capacitor of value $R^3 C^2$

Impedance Converters

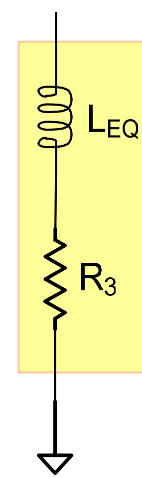


$$I_1 = \left(V_1 - \left(\frac{Z_1}{Z_1 + Z_2} \right) V_1 \right) G_3$$

$$Z_{IN} = Z_3 \left(1 + \frac{Z_2}{Z_1} \right)$$

If $Z_3 = R_3$, $Z_2 = R_2$ and $Z_1 = 1/sC$

$$Z_{IN} = R_3 + s\left(CR_2R_3\right)$$



$$L_{EQ} = CR_2R_3$$

End of Lecture 15